

2018/4 博士班資格考： 機率與統計

1. (25%) Show all details.

- (a) A random sample of n observations, X_i for $i=1,2,\dots,n$, is taken from a normal population with mean μ and variance σ^2 . Let

$$\bar{X} = \frac{X_1 + X_2 + \dots + X_n}{n}. \text{ Find the pdf of } \bar{X}.$$

- (b) Let $S^2 = \frac{\sum_{i=1}^n (X_i - \bar{X})^2}{n-1}$. Find the pdf of S^2 .

- (c) Let F be uniformly distributed with pdf, $f(x) = \begin{cases} 1/2 & , 0 \leq x \leq 2 \\ 0 & , \text{otherwise} \end{cases}$.
Let F_1 and F_2 be 2 observations of F and $G = F_1 + F_2$. Find the pdf of G .

2. (25%) If the RVs X and Y are jointly normal, then

1. Show that $f(y|x) = \frac{1}{\sigma_Y \sqrt{2\pi(1-r^2)}} \exp\left[-\frac{(y - r\sigma_Y x / \sigma_X)^2}{2\sigma_Y^2(1-r^2)}\right]$ if

X and Y are of zero mean.

2. Show that $E[Y|x] = \eta_Y + r\sigma_Y \frac{x - \eta_X}{\sigma_X}$ if X and Y have means,

η_X and η_Y , respectively.

3. (25%) Let the two random variables X and Y be joint normal with zero mean

and σ^2 . Let two random variables be $r = \sqrt{X^2 + Y^2}$ and $\theta = \tan^{-1}(\frac{Y}{X})$.

Find the joint pdf of r and θ .

4. (25%) Show all details.

- (a) Write down the pdfs of a binomial distribution, $b(x;n,p)$, and a Poisson distribution, $p(x; \mu)$.
- (b) Prove that when $n \rightarrow \infty$, $p \rightarrow 0$ and $\mu = np$ remains a constant, $b(x;n,p) \rightarrow p(x; \mu)$.