

Ph.D. Qualify Examination 2016  
Theory of Computation

- This examination is closed books.
- Please turn off your cell phones.
- Remember that there are 2 pages of the qualify examination.
- Answer all questions as possible. You may have a partial score if you answer the correct direction.

1. Deterministic Finite Acceptor (DFA) (10 pts)  
For  $\Sigma = \{a, b\}$ , construct a DFA that accepts the set consisting of:  
All strings with an even number of a's and an odd number of b's.
2. Nondeterministic Finite Acceptor (NFA) (10 pts)  
Find an NFA with four states that accepts the language  
 $L = \{a^n : n \geq 0\} \cup \{b^m a : m \geq 1\}$ .
3. Convert the nfa defined by  
 $\delta(q_0, a) = \{q_0, q_1\}$   
 $\delta(q_1, b) = \{q_1, q_2\}$   
 $\delta(q_2, a) = \{q_2\}$   
 $\delta(q_0, \lambda) = \{q_2\}$   
with initial state  $q_0$  and final state  $q_2$  into an equivalent dfa. (10 pts)
4. Please use the pumping lemma to prove that the language is not regular: (15 pts)  
 $L = \{a^n b^l a^k : k \leq n + l\}$ .
5. Show that the following grammar is ambiguous. (10 pts)  
 $S \rightarrow AB|aaB,$   
 $A \rightarrow a|Aa,$   
 $B \rightarrow b.$

6. Remove all unit-productions, all useless productions, and all  $\lambda$ -productions from the grammar:

$$S \rightarrow aA|aBB,$$

$$A \rightarrow aaA|\lambda,$$

$$B \rightarrow bB|bbC,$$

$$C \rightarrow B.$$

What language does this grammar generate? (15 pts)

7. Construct an NPDA that accepts the following language: (10 pts)

$$L = \{a^n b^{3n} : n \geq 0\}$$

8. Fill the following languages into the language hierarchy (If  $L_i$  is a regular language and also a context-free language, please fill  $L_i$  in the set of regular languages): (20 pts)

$$L_1 = \{a^3 b^n c^n : n \geq 0\},$$

$$L_2 = \{a^n b^n c^n : n \geq 0\},$$

$$L_3 = \{a^n w w^R a^n : n \geq 0, w \in \{a, b\}^*\},$$

$$L_4 = L(a^* b^*),$$

$$L_5 = \{ww : w \in \{a, b\}^*\},$$

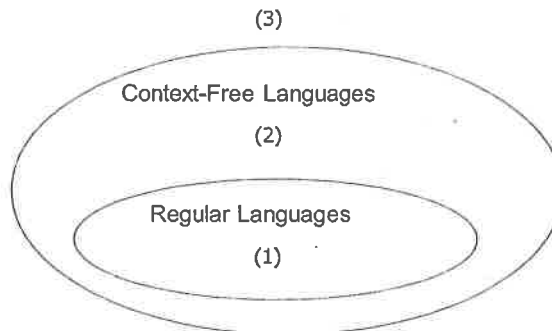
$$L_6 = \{a^{n!} : n \geq 0\},$$

$$L_7 = \{a^n b^j : n \leq j^2\},$$

$$L_8 = \{a^n b^j a^j b^n : n \geq 0, j \geq 0\},$$

$$L_9 = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\},$$

$$L_{10} = \{a^n b^m : n \geq m\}.$$



**2016 Apr. NCKU CSIE PH.D. Qualification Examination**  
**Computer Architecture**

1. With dynamic hardware prediction for reducing branch costs, what is the disadvantage of a simple 1-bit branch-prediction buffer for a branch that is almost always taken. Explain why the 2-bit prediction scheme can remedy this disadvantage. Also, explain what is correlated predictors by illustrating an example. (20 points)
2. Explain the following synchronization primitives: atomic exchange, test-and-set, and fetch-and-increment. Also, explain what is the pair of instructions, load linked (LL) and store conditional (SC) and how this pair of instructions can be used to implement atomic exchange and fetch-and-increment. (15 points)
3. The classical approach to improving cache behavior is to reduce miss rates. Please summarize the techniques that can reduce miss rates. (20 points)
4. Describe two major instruction set characteristics that can further divide general purpose register (GPR) instruction set architecture into three classes, based on whether the instruction operands are used explicitly or implicitly. And show the advantages and disadvantages of these three further divided classes. (15 points)
5. Describe what are the RAW, WAW, and WAR hazards. (15 points)
6. For the memory-hierarchy design, please answer the following questions: (Assuming the cache is  $n$ -way set associative and there are  $S = 2^s$  sets, and each block is of size  $B = 2^b$  bytes) (15 points)
  - A. Where can a block with variable  $v$  be placed in a cache? (assume  $v$  has the address  $addr$ )
  - B. How is a block with variable  $v$  is found if it is in the cache? (describe the general tag design)
  - C. Which block should be replaced on a cache miss? (make your assumption)
  - D. What happens for a write operation? (describe two basic write policies)

## OS 資格考題 (104 學年度第二學期)

1. [20%] A computer with a 32-bit address. The physical memory size is 1 GB, and each page is 4KB. A program has its code and data together fitted in the lowest 8KB and stack fitted in the highest 8KB of its virtual address space.
  - (a) [10%] What is the size of the page table if 1-level traditional page table is used? Assume that each page table entry occupies 4 bytes.
  - (b) [10%] What is the size of the inverted page table? Assume that each inverted page table entry occupies 4 bytes.
2. [20%] What is a **signal** in an UNIX system? What is a **system call** in an operating system?
3. [20%] What is the difference between a **page fault** and a **TLB miss**?
4. [15%] What is the benefit of using a **reader-writer semaphore**, compared with a traditional semaphore?
5. [15%] Please explain **processor affinity** in a multiprocessor system.
6. [10%] Describe the **many-to-1 threading model**.

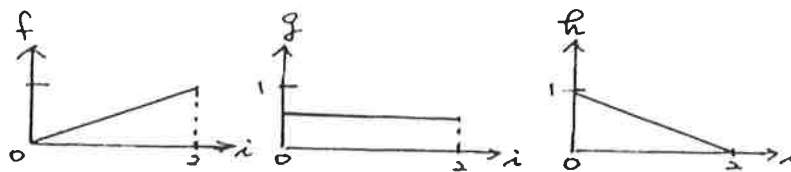
## 圖論資格考 April 15, 2016

1. (10%) Determine the Wiener index of  $n$ -vertex star.
2. (10%) Draw all the trees with vertices  $\{1,2,3,4,5,6,7\}$  that have degrees  $(3,1,2,1,3,1,1)$ .
3. (10%) Prove that a graph is bipartite if and only if it has no odd cycle.
4. (10%) For a set  $S \subseteq N$  of size  $n$ , determine the number of trees with vertex set  $S$ .
5. (10%) Show that the isomorphism relation is an equivalence relation on the set of simple graphs.
6. (10%) Prove that every closed odd walk contains an odd cycle.
7. (10%) Show that the minimum number of edges in a connected graph with  $n$  vertices is  $n-1$ .
8. (10%) Prove that the center of a tree is a vertex or an edge.
9. (10%) Let  $d_G(v)$  be the number of edges incident to  $v$  in  $G$ , except that each loop at  $v$  counts twice. Show that if  $G=(V,E)$  is a graph, then  $\sum_{v \in V} d_G(v) = 2|E|$ .
10. (10%) Present the Kruskal's algorithm for finding a minimum spanning tree.

**Digital Image Processing,  
PhD Qualification Exam,  
Department of CSIE**

April 2016

1. Please describe what the Hough transform is. (5%) Please explain why the  $(r, \theta)$  domain is used instead of the  $(a, b)$  domain in Hough transform. (5%) Please describe how to use Hough transform to detect coins with three different diameters from an image. (10%)
2. We have an image whose histogram is  $f(i)$ . Please do histogram equalization to make the histogram transferred to  $g(i)$ . (10%) then do histogram modification to make the histogram transferred to  $h(i)$ . (10%)



3. If you want to remove noises from a given image, you can usually do filtering in the spatial or frequency domains. Please describe the procedures of filtering in spatial and frequency domains. (10%) Are there differences between the two filtering methods? (5%) How can you evaluate the filtering results in signal noise ratio (give equation)? (5%)
4. During image acquisition, an image undergoes uniform linear motion in the vertical direction for a time period  $T_1$ . The direction of motion then switches to the horizontal direction for a time interval  $T_2$ . Assuming that the time it takes the image to change direction is negligible, and that the shutter opening and closing times are negligible also, give an expression for the blurring function  $H(u, v)$ .
5. If a Gaussian filter is  $G(x, y) = e^{-\frac{x^2+y^2}{2\sigma^2}}$ . If Laplacian is the sum of second derivatives of  $x$  and  $y$  variable, please show Laplacian of Gaussian (LOG) filter is

$$h(x, y) = \frac{x^2 + y^2 - 2\sigma^2}{\sigma^4} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

(10%) If the difference of Gaussian is used to approximate the LOG with  $\sigma_1 > \sigma_2$ , please show the two filters will have the same

zero crossing if  $\sigma^2 = \frac{\sigma_1^2 \sigma_2^2}{\sigma_1^2 - \sigma_2^2} \ln \left[ \frac{\sigma_1^2}{\sigma_2^2} \right]$ . (10%)

**4/2012 博士班資格考： 機率與統計 Show All Details.**

1. (20%)

Let  $X$  and  $Y$  be two random variables with uniform distribution in the interval  $[0,1]$ . Let  $Z=2X+3Y$ . Find the probability distribution of  $Z$ .

2. (20%) A pollution investigation was made upon a river based on a certain chemical substance measured in milligrams per liter. 15 samples were collected from station 1 and 20 samples were obtained from station 2. The average in station 1 is 3.5 milligrams per liter and a standard deviation being 2.5 milligrams per liter and 1.8 milligrams per liter and 1.0 milligrams per liter for station 2. Find the 95% confidence interval for the difference in the true average substance at the two stations, assuming the observations are all normally distributed.

3. (20%) Let  $X$  be a random variable and  $Y=g(X)$ . Prove that

(a)  $E[Y] = E[g(x)]$ .

(b)  $f_Y(y) = \frac{f_X(x_1)}{|g'(x_1)|} + \frac{f_X(x_2)}{|g'(x_2)|} + \dots + \frac{f_X(x_n)}{|g'(x_n)|} + \dots$ , where

$$y = g(x_1) = g(x_2) = \dots$$

4. (20%)

(a) State the conditions of a stochastic process which is called wide-sense stationary.

(b) If  $x(t)$  is a W.S.S. stochastic process, show that  $E\{[x(t+T) - x(t)]^2\} = 2\{R(0) - R(T)\}$ .

5. (20%) A random variable  $X$  has the distribution as

$$P(X = k) = pq^k, \quad k = 0, 1, \dots$$

Find  $\Gamma(z)$  and use this to find the mean and variance.

## 無線通訊與行動計算 資格考

1. Please explain the below terms in details. [25 %]
  - A. Poisson Distribution
  - B. Delay Spread
  - C. TDMA/ FDMA/ CDMA
  - D. Reuse Distance
  - E. Doppler Shift
2. Consider a cruise boat with two passengers. Each passenger will make 3 calls per hour with each call of 4-minute duration. There is only one telephone set on the boat. Please calculate the probability of the phone being occupied by one person while the other person wishes to make a call. [20%]
  - A. Please draw the Markov Chain,
  - B. Please calculate the blocking probability.
3. Please describe the following protocol in Pseudo code. [20%]
  - A. Aloha
  - B.  $p$ -persistent CSMA
  - C. CSMA/CD
  - D. CSMA/CA
4. Please calculate the maximum transmission rate for a transmission system when the delay spread is 5 ns (nano-seconds). [ 7 %]
5. Please explain the following terms in details. [16 %]
  - A. HLR and VLR (of GSM)
  - B. SGSN and GGSN
  - C. Backoff mechanism of IEEE 802.11
  - D. Hidden terminal problem
6. Define the first-meter path loss as the received signal strength (in dB) when the receiver stands one meter away from the transmitter. Now, consider the case when the first-meter path loss is 20dB . Please calculate the free-space path loss for a receiver if the distance between the transmitter and receiver is [12 %]
  - A. 10 meters,
  - B. 100 meters,
  - C. 1 KM.



## Biostatistics test for Qualification of PhD candidate

### Problems and discussions (申論題)

1. A biomedical researcher found his (her) experimental results with statistical test was  $P = 0.051$ . To our common understanding, this  $p$  value shows there is no statistical significance on the data, please answer and discuss the following questions
  - a) How do you explain this  $p$  value properly and encourage him/her?
  - b) How do you help him/her?
2. What's difference between standard (標準差) deviation and standard error/standard error of the mean (SEM, 標準誤)? Also, please explain more about confidence interval for the mean?
3. What are the typical steps in a Statistical test of hypothesis?
4. Please explain the concepts and relationship between type I error, type II error and the power of the test. How are these factors relevant to sample size?
5. What are the major concerns and proper statistical testing methods of two samples used for continuous (under parametric approach) and categorical data types?

# Discrete-Time Signal Processing 資格考

April 2016

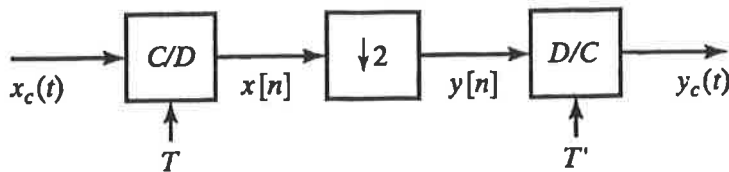
1. (20%) Determine if the systems described by the following input-output equations are (1) linear, (2) stable, and (3) causal.
- (a)  $y[n] = 4x[n] + 5$
- (b)  $y[n] = \log(x[n])$

**Justify your answer.**

2. (20%) In the following figure,  $x[n] = x_c(nT)$  and  $y[n] = x[2n]$
- (a) Assume that  $x_c(t)$  has a Fourier transform such that  $X_c(j\Omega) = 0, |\Omega| > 2\pi(100)$ . What value of  $T$  is required so that

$$X(e^{j\omega}) = 0, \quad \frac{\pi}{2} < |\omega| \leq \pi?$$

- (b) How should  $T'$  be chosen so that  $y_c(t) = x_c(t)$ ?



3. (20%) Consider a right-sided sequence  $x[n]$  with z-transform

$$X(z) = \frac{2z^2 - z}{2z^2 + \frac{3}{2}z + \frac{1}{4}}$$

Determine the inverse z-transform using each of the following methods

4. (20%) A discrete-time causal LTI system has the system function

$$H(z) = \frac{(1 + 0.2z^{-1})(1 - 9z^{-2})}{(1 + 0.81z^{-2})}$$

- (a) Is the system stable?
- (b) Determine expressions for a minimum-phase system  $H_l(z)$  and an all-pass system  $H_{ap}(z)$  such that

$$H(z) = H_l(z) H_{ap}(z)$$

5. (20%). Suppose that we wish to design a highpass filter satisfying the following specification:

$$-0.04 < |H(e^{j\omega})| < 0.04, \quad 0 \leq |\omega| \leq 0.2\pi,$$

$$0.995 < |H(e^{j\omega})| < 1.005, \quad 0.3\pi \leq |\omega| \leq \pi.$$

The filter will be designed using the bilinear transformation and  $T=2$  ms with a prototype continuous-time filter. State the specification that should be used to design the prototype continuous-time filter to ensure that the specification for the discrete-time filter are met. The following equations are for your reference :

$$s = \frac{2}{T} \left( \frac{1 - z^{-1}}{1 + z^{-1}} \right), \quad \Omega = \frac{2}{T} \tan(\omega/2), \quad \omega = 2 \arctan(\Omega T / 2)$$

## Algorithms 資格考 April 2016

1. (10%) Give formal definitions of  $\Theta(g(n))$ ,  $O(g(n))$ , and  $\Omega(g(n))$ .
2. (10%) Give asymptotic upper and lower bound for  $T(n) = \sqrt{n}T(\sqrt{n}) + n$ . Assume that  $T(n)$  is constant for sufficiently small  $n$ . Make your bounds as tight as possible.
3. (10%) If possible, use the master method to solve  $T(n) = 27T(\frac{n}{3}) + \Theta(\frac{n^3}{\lg n})$ .
4. (20%) Show that any comparison sort algorithm requires  $\Omega(n \lg n)$  comparisons in the worst case.
5. (10%) What are the minimum and maximum numbers of elements in a heap of height  $h$ ?
6. (10%) Present the quick sort algorithm and analyze the complexity.
7. (10%) The matrix-chain multiplication problem can be stated as follows: Given a chain  $\langle A_1, A_2, \dots, A_n \rangle$  of  $n$  matrices, where for  $i=1, 2, \dots, n$ , matrix  $A_i$  has dimension  $p_{i-1} \times p_i$ , fully parenthesize the product  $A_1 A_2 \dots A_n$  in a way that minimizes the number of scalar multiplications. Suppose that you have 6 matrices:  $A_1$  has dimension  $30 \times 35$ ,  $A_2$  has dimension  $35 \times 15$ ,  $A_3$  has dimension  $15 \times 5$ ,  $A_4$  has dimension  $5 \times 10$ ,  $A_5$  has dimension  $10 \times 20$ ,  $A_6$  has dimension  $20 \times 25$ . Please calculate the minimum number of scalar multiplications.
8. (20%) (a) (10%) Determine which one of the 0-1 knapsack problem and the fractional knapsack problem cannot be solved using the greedy strategy? (b) (10%) Give an example to explain that.